zwyczajnej *(Pinus silvestris* L.). (Variation in recent and fossil cones of *Pinus silvestris* L.). Floristica et Geobotanica 7 (1): 97–160 (1960). — STASZKIEWICZ, J.: Biometrie studies on the cones of *Pinus silvestris* L. growing in Hungary. Acta Botanica 7: 451—466 (1961). — STASZKIEWICZ, J.: Badania biometryczne nad zmiennoscia szyszek sosny szwyczajnej *(Pinus silvestris* L.) w Masymywie Centralnym we Francji. (Recherches biométriques sur la variabilite des cones du pin sylvestre *(Pinus silvestris* L.) du Massif central en France). Floristica et Geobotanica 9 (2): 175–187 (1962). — STASZKIEWICZ, J.: Sosna zwyczajna, *Pinus silvestris* L., systematyka i zmiennosc. (Scotch pine, systematics and variability). Zaklad Dendrologii i Arboretum Kornickie, Polskiej Akad. Nauk, Nasze Drzewa Lesne, Tom I, 55–77 (1970). — STEINBECK, K.: Foliar mineral accumulation by several Scotch pine *(Pinus sylvestris* L.) provenances. Ph. D. thesis, Mich. State Univ. 115 pp. (1965). — STEINER, K.: Genetic differences in resistance of Scotch pine to the eastern pineshoot borer. The Great Lakes Entomologist 7: 103–107 (1974). — SVOBODA, P.: Lesni dreviny a jejich porosty. Cast 1. (Forest trees and shrubs and their stands, part 1). Statni Zem. Nakl. Praha. 411 pp. (1953). — TOBOLSKI, 3. and HANOVER, J. W.: Genetic variation in the monoterpenes of Scotch pine. Forest Sci. 17: 293–299 (1971). — WRIGHT, J. W. and BALDWIN, H. I.: The Interantional Union Scotch pine provenance test in New Hampshire. Silvae Genetica 6: 2–14 (1957). — WRIGHT, J. W. and BULL, W. I.: Geographie variation in Scotch pine. Silvae Genetica 12: 1–25 (1963). — WRIGHT, J. W., LEMMIEN, W. L., BRIGHT, J. N., DAY, M. W. and SAJDAK, R. L.: Scotch pine varieties for Christmas tree and forest planting in Michigan. Mich. Agri. Exper. Sta. Res. Rpt. 293. 15 pp. (1976). — WRIGHT, J. W., PAULEY, S. S., POLK, R. B., JOKELA, J. J. and READ, R. A.: Performance of Scotch pine varieties in the north central region. Silvae Genetica 15: 101–110 (1966). — WRIGHT, J. W. and WILSON, L. F.: Genetic differences in Scotch pine resistance to pine root collar weevil. Mich. Agri. Exper. Sta. Res. Rpt. 159. 5 pp. (1972). — WRIGHT, J. W., WILSON, L. F. and BRIGHT, J. N.: Genetic variatiton in resistance of Scotch pine to Zimmerman pine moth. The Great Lakes Entomologist 8: 231–236 (1975). — WRIGHT, J. W., WILSON, L. F. and RANDALL, W. K.: Differences among Scotch pine varieties in susceptibility to European pine sawfly. Forest Sci. 13: 175–181 (1967).

Two-Level Diallel Cross Experiments
II. Incomplete Environmental Designs*)

By B. COX and K. HINKELMANN

Department of Statistics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061 (U.S.A.)

(Received June 76)

1. Introduction

As a possible experimental procedure for studying and evaluating interpopulation crosses (to be understood in a very general sense), HINKELMANN (1974) proposed a two-level diallel cross experiment. The mating design for this experiment, involving m populations and n individuals per population, calls for $m(m-1)/2$ "population crosses" and n^2 "individual crosses" for each population cross. This may lead to a rather large number of crosses and hence to a large number of seedlings that have to be grown.

It is here then that we have to give attention to the second design aspect for a genetic experiment: the environmental design (for a more general discussion of design aspects we refer to HINKELMANN, 1975). In this paper we propose a practical and workable plan for the layout of the field experiment for the two-level diallel cross. This environmental design is an incomplete block design of a special type reflecting the particular structure of the underlying mating design.

2. The environmental design

As pointed out earlier, in order to accommodate the generally large number of crosses and adequately control environmental effects one will need to use some form of incomplete block design. Rather than using the most general approach to this end, *i.e.*, identify the crosses with treatments and use any available incomplete block design, we shall consider a design that takes into account the fact that the crosses have a two-level structure. This allows us to affect a reduction in block size at both levels either separately or simultaneously. From the point of view of simplicity of experimentation it seems fairly obvious here to consider a reduction in block size through the population crosses as follows.

For a two-level diallel in a randomized complete block design each block would consist of $m(m-1)n^2/2$ crosses, *i.e.*, n^2 individual crosses for each of the $t = m(m-1)/2$ population crosses. One way to reduce the block size is to reduce the number of population crosses occurring in a block. We achieve this by relating the t population crosses to the treatments of an incomplete block design as illustrated in Fig. 1 for $m = 4$. This triangular array tells us that, for example, population cross P, X P, corresponds to treatment 5, etc. We now look for an appropriate balanced incomplete (BIB) or partially balanced incomplete block (PBIB) design with blocks of size k as listed in COCHRAN and COX (1957) or CLATWORTHY (1973). Each block of the resulting environment-

	P_1	P_2	P_3	P_4
P_1	*	1	2	3
P_2		*	4	5
P_3			*	6
P_4				*

Fig. 1. — Correspondence between population crosses and treatments of an incomplete block design.

*) Herrn Professor Dr. WOLFGANG LANGNER zu seinem 70. Geburtstag gewidmet.

Table 1. — Example of an incomplete environmental design (only the population crosses are indicated).

Block	Population crosses
1	$P_1 \times P_2$, $P_1 \times P_3$, $P_2 \times P_4$
2	$P_1 \times P_2$, $P_1 \times P_3$, $P_3 \times P_4$
3	$P_1 \times P_2$, $P_1 \times P_4$, $P_2 \times P_3$
4	$P_1 \times P_2$, $P_1 \times P_4$, $P_3 \times P_4$
5	$P_1 \times P_2$, $P_2 \times P_3$, $P_2 \times P_4$
6	$P_1 \times P_3$, $P_1 \times P_4$, $P_2 \times P_3$
7	$P_1 \times P_3$, $P_1 \times P_4$, $P_2 \times P_4$
8	$P_1 \times P_3$, $P_2 \times P_3$, $P_3 \times P_4$
9	$P_1 \times P_4$, $P_2 \times P_4$, $P_3 \times P_4$
10	$P_2 \times P_3$, $P_2 \times P_4$, $P_3 \times P_4$

al design for the two-level diallel cross contains kn^2 plots with seedlings from kn^2 crosses rather than from tn^2 crosses as for a randomized complete block design.

Table 1 gives an example for $m = 4$ populations, *i.e.*, $t = 6$ population crosses, using BIB plan 11.4 of COCHRAN and COX (1957) together with the correspondence set up in *Fig. 1*. Each of the $k = 3$ "plots" per block contains seedlings from n^2 individual crosses.

So far we have described the procedure for finding an appropriate incomplete environmental design in general terms, using any available incomplete block design. However, AGGARWAL (1973) has pointed out (see also HINKELMANN, 1975) that PBIB designs with a triangular association scheme (T-PBIB) are particularly well suited for diallels of Type II (HINKELMANN and STERN, 1960) as they relate in a natural way, through their analysis, to the concepts of general and specific combining abilities (associated with the population crosses in our case). For this reason we shall confine ourselves in the following sections to T-PBIB designs as the generators of incomplete environmental designs (see also Section 4).

3. Statistical analysis

Let $Y_{(ik)(jl)z}$ denote the observation for the offspring (seedling) of the mating $I_{ik} \times I_{jl}$ (where I_{ik} refers to the kth individual (tree) in P_i, the ith population) in block z of the environmental design. The basic model underlying the analysis is of the form

$$Y_{(ik)(jl)z} = \mu + c_{(ik)(jl)} + b_z + \varepsilon_{(ik)(jl)z} \quad (1)$$

where

$$c_{(ik)(jl)} = G_i + G_j + S_{ij} + g_{ik} + g_{jl} + s_{(ik)(jl)}. \quad (2)$$

Where $c_{(ik)(jl)}$ refers to the effect of the cross $I_{ik} \times I_{jl}$, and b_z is the effect of the zth block. For the meaning and definition of the other parameters we refer to HINKELMANN (1974). Since we are using an incomplete block design only certain combinations (ik) (jl)z occur based on the generating T-PBIB design as outlined in Section 2.

Estimates of the combining abilities in (2) are obtained as linear combinations of the estimates of the $c_{(ik)(jl)}$ from (1), which in turn are the intra-block estimates, denoted by $\hat{c}_{(ik)(jl)}$, of the treatment effects, i.e., cross effects for the underlying incomplete block design.

Let (t, b, k, r, λ_1, λ_2) be the parameters of the generating T-PBIB design and

$$\Theta_1 = r + (m-4)\lambda_1 - (m-3)\lambda_2$$
$$\Theta_2 = r - 2\lambda_1 + \lambda_2.$$

Further let

$$Q_{(ik)(jl)} = Y_{(ik)(jl).} - \cdot \frac{1}{kn^2} \sum_{z=1}^{b} n_{(i,j)z} B_z$$

be the adjusted ($I_{ik} \times I_{jl}$) — cross total, where

$$n_{(i,j)z} = \begin{cases} 1 & \text{if } P_i \times P_j \text{ occurs in block z} \\ 0 & \text{otherwise} \end{cases}$$

and B_z is the zth block total. Then

$$\hat{c}_{(ik)(jl)} = \frac{1}{r} Q_{(ik)(jl)} - a_1 S_1 (Q_{(ik)(jl)}) - a_2 S_2 (Q_{(ik)(jl)}) \quad (3)$$

where

$$a_1 = \frac{1}{(m-2)n^2r} [(1-\Theta_1/rk)^{-1} + (m-3)(1-\Theta_2/rk)^{-1} - (m-2)],$$

$$a_2 = \frac{1}{(m-2)n^2r} [2(1-\Theta_1/rk)^{-1} + (m-4)(1-\Theta_2/rk)^{-1} - (m-2)],$$

$$S_1 (Q_{(ik)(jl)}) = \sum_{\substack{j' \neq j \\ j' > i}} \sum_{k',l'} Q_{(ik')(j'l')} + \sum_{\substack{j' \neq j \\ j' < i}} \sum_{k',l'} Q_{(j'l')(ik')} + \sum_{\substack{i' \neq i \\ i' < j}} \sum_{k',l'} Q_{(i'k')(jl')} + \sum_{\substack{i' \neq i \\ i' > j}} \sum_{k',l'} Q_{(jl')(i'k')}$$

= sum of all those adjusted cross totals that involve either P_i or P_j, but not both,

$$S_2 (Q_{(ik)(jl)}) = \sum_{\substack{i',j' \\ i' < j' \\ (i',j') \neq (i,j)}} \sum_{k',l'} Q_{(i'k')(j'l')}$$

= sum of all those adjusted cross totals that do not involve P_i or P_j.

Using the dot notation

$$\hat{c}_{(ik)(..)} = \sum_{j<i} \sum_{l} \hat{c}_{(jl)(ik)} + \sum_{j>i} \sum_{l} \hat{c}_{(ik)(jl)}$$

$$\hat{c}_{(i.)(j.)} = \sum_{k,l} \hat{c}_{(ik)(jl)}$$

$$\hat{c}_{(i.)(..)} = \sum_{k} \hat{c}_{(ik)(..)}$$

we can then express the estimates of the combining abilities as follows:

$$\hat{G}_i = \hat{c}_{(i.)(..)}/(m-2)n^2$$
$$\hat{S}_{ij} = \hat{c}_{(i.)(j.)}/n^2 - (\hat{c}_{(i.)(..)} + \hat{c}_{(j.)(..)})/(m-2)n^2$$
$$\hat{g}_{ik} = \hat{c}_{(ik)(..)}/(m-1)n - \hat{c}_{(i.)(..)}/(m-1)n^2$$
$$\hat{s}_{(ik)(jl)} = \hat{c}_{(ik)(jl)} - (\hat{c}_{(ik)(..)} + \hat{c}_{(jl)(..)})/(m-1)n + (\hat{c}_{(i.)(..)} + \hat{c}_{(j.)(..)})/(m-1)n^2 - \hat{c}_{(i.)(j.)}/n^2.$$

Table 2. — Comparisons between combining abilities and their variances.

Comparison	Variance
(1) $\hat{G}_i - \hat{G}_{i'}$	$2\sigma^2/[(m-2)n^2r(1-\Theta_1/rk)]$
(2) $\hat{S}_{ij} - \hat{S}_{i'j}$	$2(m-3)\sigma^2/[(m-2)n^2r(1-\Theta_2/rk)]$
$\hat{S}_{ij} - \hat{S}_{i'j'}$ ($i \neq i', j \neq j'$)	$2(m-4)\sigma^2/[(m-2)n^2r(1-\Theta_2/rk)]$
(3) $\hat{g}_{ik} - \hat{g}_{ik'}$	$2\sigma^2/(m-1)nr$
(4) $\hat{s}_{(ik)(j\ell)} - \hat{s}_{(ik)(j\ell')}$	$2[(m-1)n-1]\sigma^2/(m-1)nr$
$\hat{s}_{(ik)(j\ell)} - \hat{s}_{(ik')(j\ell')}$ ($k \neq k', \ell \neq \ell'$)	$2[(m-1)n-2]\sigma^2/(m-1)nr$

Comparisons between combining abilities and associated variances are given in *Table 2*. Also, information about comparisons of average cross performance (at the population as well as at the individual level) is given in *Table 3*. For remarks concerning the nature of σ^2 and its estimate we refer to HINKELMANN (1974). Any modifications should follow from looking at the analysis of variance table and the expected mean squares under the various models, as presented in Tables 4 and 5, respectively.

The analysis, as presented here, assumes that each field plot contains one seedling from one cross. The modification for $p\,(>1)$ seedlings per cross per field plot follows the familiar rules.

4. Generator T-PBIB designs

Having described (Section 2) a general method of obtaining one type of incomplete environmental design for the two-level diallel cross experiment, and having given (Section 3) the analysis for such designs generated by T-PBIB designs, we shall comment in this section on some aspects and properties of available T-PBIB designs.

Table 3. — Comparisons of average performance.

Type of comparison	Estimator	Variance
(5) $G_i - G_{i'}$	$(\hat{c}_{(i\cdot)(\cdot\cdot)} - \hat{c}_{(i'\cdot)(\cdot\cdot)})/(m-2)n^2$	$2\sigma^2/(m-2)n^2r(1-\Theta_1/rk)$
(6) $(G_i+G_j+S_{ij})$ $-(G_{i'}+G_{j'}+S_{i'j'})$	$(\hat{c}_{(i\cdot)(j\cdot)} - \hat{c}_{(i'\cdot)(j'\cdot)}/n^2$	$4\sigma^2/(m-2)n^2r(1-\Theta_1/rk)$ $+2(m-4)\sigma^2/(m-2)n^2r(1-\Theta_2/rk)$ for $i\neq i', j\neq j'$ $2\sigma^2/(m-2)n^2r(1-\Theta_1/rk)$ $+2(m-3)\sigma^2/(m-2)n^2r(1-\Theta_2/rk)$ for $i=i', j\neq j'$
(7) $(G_i+g_{ik})-(G_{i'}+g_{i'k'})$	$(\hat{c}_{(ik)(\cdot\cdot)} - \hat{c}_{(i'k')(\cdot\cdot)})/(m-1)n$ $+(\hat{c}_{(i\cdot)(\cdot\cdot)} - \hat{c}_{(i'\cdot)(\cdot\cdot)})/(m-1)(m-2)n^2$	$2\sigma^2/(m-2)n^2r(1-\Theta_1/rk)$ $+2(n-1)\sigma^2/(m-1)n^2r$ for $i\neq i'$ $2\sigma^2/(m-1)nr$ for $i=i'$
(8) $(G_i+G_j+S_{ij}+g_{ik}$ $+g_{j\ell}+s_{(ik)(j\ell)})$ $-(G_{i'}+G_{j'}+S_{i'j'}+g_{i'k'}$ $+g_{j'\ell'}+s_{(i'k')(j'\ell')})$	$\hat{c}_{(ik)(j\ell)} - \hat{c}_{(i'k')(j'\ell')}$	$4\sigma^2/(m-2)n^2r(1-\Theta_1/rk)+2(m-4)\sigma^2/(m-2)n^2r(1-\Theta_2/rk)$ $+2(n^2-1)\sigma^2/n^2r$ for $i\neq i', j\neq j'$ $2\sigma^2/(m-2)n^2r(1-\Theta_1/rk)+2(m-3)\sigma^2/(m-2)n^2r(1-\Theta_2/rk)$ $+2(n^2-1)\sigma^2/n^2r$ for $i=i', j\neq j'$ $2\sigma^2/r$ for $i=i', j=j'$

Table 4. — Analysis of variance.

Source	d.f.	Sum of squares	Mean square
Blocks ignoring crosses	$b-1$	$\frac{1}{kn^2}\sum_{z=1}^{b} B_z^2 - \frac{2G^2}{m(m-1)n^2r}$ *)	
Crosses eliminating blocks	$\frac{m(m-1)}{2}n^2-1$	$\sum_{i<j}\sum_{k,\ell} \hat{c}_{(ik)(j\ell)} Q_{(ik)(j\ell)}$	
G	$m-1$	$\frac{(m-2)n^2}{k}(rk-\Theta_1)\sum_{i=1}^{m}\hat{G}_i^2$	MS(G)
S	$m(m-3)/2$	$\frac{n^2}{k}(rk-\Theta_2)\sum_{i,j;\,i<j}\hat{S}_{ij}^2$	MS(S)
g	$m(n-1)$	$(m-1)nr\sum_{i=1}^{m}\sum_{k=1}^{n}\hat{g}_{ik}^2$	MS(g)
s	$\frac{m(m-1)}{2}(n^2-1)-m(n-1)$	$r\sum_{i,j;\,i<j}\sum_{k,\ell}\hat{s}_{(ik)(j\ell)}^2$	MS(s)
Error	$\frac{m(m-1)}{2}n^2(r-1)-b+1$	by subtraction	MS(ε)
Total	$\frac{m(m-1)}{2}n^2r-1$	$\sum_{i,j;\,i<j}\sum_{k,\ell}\sum_{z} Y_{(ik)(j\ell)z}^2 - \frac{2G^2}{m(m-1)n^2r}$	

*) G = grand total

Table 5. — Expected mean squares.

Mean square	Model (i)	Model (ii)	Model (iii)
MS(G)	$\sigma_\varepsilon^2 + \frac{(m-2)n^2}{m-1}\phi_1 \sum_i G_i^2$	$\sigma_\varepsilon^2 + K_2\sigma_s^2 + nK_1\sigma_g^2 + n^2K_2\sigma_S^2 + n^2K_1\sigma_G^2$	$\sigma_\varepsilon^2 + K_2\sigma_s^2 + nK_1\sigma_g^2 + \frac{(m-2)n^2}{m-1}\phi_1 \sum_i G_i^2$
MS(S)	$\sigma_\varepsilon^2 + \frac{2n^2}{m(m-3)}\phi_2 \sum_{i<j} S_{ij}^2$	$\sigma_\varepsilon^2 + K_4\sigma_s^2 + nK_3\sigma_g^2 + n^2K_4\sigma_S^2 + n^2K_3\sigma_G^2$	$\sigma_\varepsilon^2 + K_4\sigma_s^2 + nK_3\sigma_g^2 + \frac{2n^2}{m(m-3)}\phi_2 \sum_{i<j} S_{ij}^2$
MS(g)	$\sigma_\varepsilon^2 + \frac{(m-1)nr}{m(n-1)} \sum_i \sum_k g_{ik}^2$	$\sigma_\varepsilon^2 + r\sigma_s^2 + (m-1)\, nr\sigma_g^2$	$\sigma_\varepsilon^2 + r\sigma_s^2 + (m-1)\, nr\sigma_g^2$
MS(s)	$\sigma_\varepsilon^2 + \frac{r}{d} \sum_{i<j} \sum_{k,\ell} s_{(ik)(j\ell)}^2$	$\sigma_\varepsilon^2 + r\sigma_s^2$	$\sigma_\varepsilon^2 + r\sigma_s^2$
MS(ε)	σ_ε^2	σ_ε^2	σ_ε^2

$$\phi_1 = r - \Theta_1/k$$

$$\phi_2 \quad r - \Theta_2/k$$

$$d = \frac{m(m-1)}{2}(n^2-1) - m(n-1)$$

$$K_1 = \frac{m}{(m-2)\phi_1 k^2}\{(m-1)[rk-r-(m-2)\lambda_1]^2 + [rk-r-3(m-2)\lambda_1 - (m-3)(m-2)\lambda_2]^2\}$$

$$K_2 = \frac{m}{(m-2)\phi_1 k^2}\{[rk-r-(m-2)\lambda_1]^2 + \frac{m-2}{2}[2\lambda_1 + (m-3)\lambda_2]^2\}$$

$$K_3 = \frac{m-1}{(m-3)(m-2)^2\phi_2}[2\phi_1 - \frac{m}{k}(2\lambda_1 + (m-3)\lambda_2)]^2$$

$$K_4 = (m^2-6m+12)\phi_2^2 - \frac{2m}{k}\phi_2[4\lambda_1+(m-4)\lambda_2] + \frac{m(m-1)}{2k^2}[4\lambda_1+(m-4)\lambda_2]^2$$

(i) There exist no T-PBIB designs for $m = 4$, i.e., $t = 6$. We propose to use instead BIB designs as the generating designs. The analysis as outlined in the previous section can still be applied as now $\lambda_1 = \lambda_2 = \lambda$, where λ is a parameter of the BIB design; also $\Theta_1 = \Theta_2 = r - \lambda$, and hence $a_1 = a_2$ in (3).

(ii) From Table 2 we see that

$$\text{Var}(\hat{G}_i - \hat{G}_{i'}) = 2\sigma^2/(m-2)n^2r(1-\Theta_1/rk)$$

and

$$\text{Var}(\hat{S}_{ij} - \hat{S}_{i'j'}) = \begin{cases} 2(m-3)\sigma^2/(m-2)n^2r(1-\Theta_2/rk) & (i \neq i', j = j') \\ 2(m-4)\sigma^2/(m-2)n^2r(1-\Theta_2/rk) & (i \neq i', j \neq j') \end{cases}$$

Compared to the same expressions for the randomized complete block design as given by Hinkelmann (1974) we see that they differ by a factor $1/(1-\Theta_1/rk)$ and $1/(1-\Theta_2/rk)$, respectively. The efficiences for comparisons of general and specific combining abilities at the population level using an incomplete environmental design (generated from a T-PBIB design) relative to those using a complete environmental design are then

$$E_G = 1 - \Theta_1/rk$$

and

$$E_S = 1 - \Theta_2/rk,$$

respectively. Hence the loss of information (assuming equal σ^2 for both types of designs) is given by Θ_1/rk and Θ_2/rk, respectively.

This association between Θ_1 and the G's and between Θ_2 and the S's is a characteristic feature of T-PBIB designs. For this reason we have restricted ourselves to T-PBIB designs (as alluded to in Section 2). It enables the experimenter to identify the most appropriate design, subject only to the constraint of the size of the experiment. For example, if it is desired to incur as little loss of information about the G_i's as possible one should choose among competing designs the one that has the smallest Θ_1/rk, zero if possible. Also, from previous remarks it is obvious that a BIB design will lead to the same loss of information for G_i's and S_{ij}'s. Incidentally, no loss of information occurs for the g_{ik} and $s_{(ik)(jl)}$.

Table 6. — BIB and T-PBIB designs.

Type	m	t	k	r	b	λ_1	λ_2	Θ_1	Θ_2	E_G	E_S
BIB 11.3	4	6	2	5	15	1	1	4	4	.60	.60
BIB 11.4	4	6	3	5	10	2	2	3	3	.80	.80
BIB 11.5	4	6	3	10	20	4	4	6	6	.80	.80
BIB 11.14	5	10	2	9	45	1	1	8	8	.56	.56
T 1	5	10	2	6	30	1	0	7	4	.42	.67
T 2	5	10	2	3	15	0	1	1	4	.83	.33
T 3	5	10	2	6	30	0	2	2	8	.83	.33
T 4	5	10	2	9	45	0	3	3	12	.83	.33
BIB 11.15	5	10	3	9	30	2	2	7	7	.74	.74
T 9	5	10	3	3	10	1	0	4	1	.56	.89
T 10	5	10	3	6	20	2	0	8	2	.56	.89
T 11	5	10	3	9	30	3	0	12	3	.56	.89
T 12	5	10	3	6	20	1	2	3	6	.83	.67
T 13	5	10	3	9	30	1	4	2	11	.93	.59
T 5	6	15	2	8	60	1	0	10	6	.38	.63
T 6	6	15	2	6	45	0	1	3	7	.75	.42
BIB 11.24	6	15	3	7	35	1	1	6	6	.71	.71
T 14	6	15	3	4	20	1	0	6	2	.50	.83
T 15	6	15	3	8	40	2	0	12	4	.50	.83
T 16	6	15	3	3	15	0	1	0	4	1.00	.56
T 17	6	15	3	6	30	0	2	0	8	1.00	.56
T 18	6	15	3	10	50	1	2	6	10	.80	.67
T 19	6	15	3	9	45	0	3	0	12	1.00	.56
T 7	7	21	2	10	105	1	0	13	8	.35	.60
T 8	7	21	2	10	105	0	1	6	11	.70	.45
BIB 11.33	7	21	3	10	70	1	1	9	9	.70	.70
T 20	7	21	3	5	35	1	0	8	3	.47	.80
T 21	7	21	3	10	70	2	0	16	6	.47	.80
T 22	7	21	3	10	70	0	2	2	12	.93	.60

(iii) For practical purposes, BIB and T-PBIB designs with block size k = 2 or 3 will generally be the most appropriate, since otherwise the overall block size (kn^2) may still become too large.

In *Table 6* we present a list of such designs together with their parameters. The numbers of the BIB designs refer to the plans given in Cochran and Cox (1957); the T-PBIB designs are labeled as in Clatworthy (1973). These designs are to be used in connection with the correspondence between treatments and population crosses as outlined in Section 2.

Summary

A general procedure for generating a certain type of incomplete environmental design for a two-level diallel cross experiment has been discussed. It is based on a correspondence between the treatments of an incomplete block design and the population crosses. The analysis of such designs is given for triangular PBIB designs as generating designs. A list of potentially useful triangular PBIB designs has been included.

Key words: Incomplete design, diallel, two-level diallel cross, triangular PBIB designs, combining abilities.

Zusammenfassung

In dieser Arbeit wird eine allgemeine Methode zur Herleitung gewisser unvollständiger Versuchspläne für ein zweistufiges dialleles Kreuzungsexperiment vorgeschlagen. Diese Methode beruht auf einer Zuordnung der Verfahren eines unvollständigen Blockversuchsplanes und den Populations-Kreuzungen. Die Auswertung solcher Versuchspläne wird angegeben für den Fall, daß „Triangular PBIB"-Pläne als Erzeuger verwendet werden. Für die praktische Anwendung nützliche Pläne sind in einer Tabelle zusammengestellt.

References

Aggarwal, K. R.: Analysis of L_i(s) and triangular designs. Unpublished (1973). — Aggarwal, K. R.: Balanced diallel experiments. Unpublished (1973). — Clatworthy, W. H.: Tables of two-associate-class partially balanced designs. Nat. Bur. Stand. (U.S.), Applied Mathematics Series 63 (1973). — Cochran, W. G. and Cox, G. M.: Experimental designs. New York, 1957. — Hinkelmann, K.: Two-level diallel cross experiments. Silvae Genetica 23, 18—22 (1974). — Hinkelmann, K.: Design of genetical experiments. In: A survey of statistical design and linear models, J. N. Srivastava, ed. Amsterdam, 1975. — Hinkelmann, K. and Stern, K.: Kreuzungspläne zur Selektionszüchtung bei Waldbäumen. Silvae Genetica 9, 121—133 (1960).

Inbreeding Douglas Fir to the S_3 Generation[1])

By A. L. Orr-Ewing

Research Division, British Columbia Forest Service, Canada

(Received July 1976)

Introduction

Douglas fir, *Pseudotsuga menziesii* (Mirb.) Franco, is a species remarkable for its adaptability to such widely different forms of breeding as racial crossing and inbreeding. As regards the former, there is no incompatibility barrier to successful crosses being made between trees growing in completely different environments and separated by many thousands of kilometres. In 1968, viable crosses were made between trees from Mexico and British Columbia, in one instance the two sources were separated by more than 25° of latitude. The progenies, moreover, are now growing vigorously at Cowichan Lake in Southern Vancouver Island. There would appear to be no limit to racial crossing within the entire range of Douglas fir in North America.

At the other extreme is inbreeding and although the Douglas fir was once considered self-sterile, S_3 inbreds have been raised from self-pollinations made on two trees in 1952 and 1954. This inbreeding program has been of necessity on a small scale but, on the other hand, has had the advantage that every pollination was made in person so that the same precautions against contamination have been taken throughout the whole period.

Inbreeding from the S_0 to the S_3 generation

The two S_0 trees, 2 and 11 were both under 25 years at the time of pollination, the former being located near Vancouver on the British Columbian mainland and the latter from Cowichan Lake in Southern Vancouver Island. *Table 1* shows the successive pollinations from the S_0 to the S_3 generation more than 20 years later. The trees used over this period are illustrated in *Figures 1—4* and *5—8*, the photographs being taken in 1976 with the exception of *1* and 5 which were photographed in 1954.

Table 1. — Result of Pollination from the S_0 to the S_3 generation, 1952—1975.

Tree	Pollin. year	Cone no.	Seed per cone	Germinants per cone	Tree	Pollin. year	Cone no.	Seed per cone	Germinants per cone
$S_0$2	1952	15	26.4	3.7	$S_0$11	1954	56	60.8	7.7
$S_1$2.21	1962	2	59.0	4.0	$S_1$11.32	1962	13	46.6	2.3
$S_1$2.21	1966	211	38.1	12.1	$S_1$11.32	1968	110	57.7	0.8
$S_1$2.21	1968	73	44.0	0.6	$S_1$11.32	1970	27	68.1	5.8
$S_1$2.21	1970	80	67.4	18.6	$S_2$11.32.6	1971	74	46.3	0.05
$S_2$2.21.23	1975	96	56.1	0.02	$S_2$11.32.6	1973	55	57.8	0.03

[1] This paper is dedicated to Dr. W. Langner on the occasion of his 70th birthday.